Non-invertible symmetries act locally by quantum operations

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joint work with Yuji Tachikawa

[arXiv:2403.20062[hep-th]]

Self Introduction

Masaki Okada

- second year PhD student of physics.
- interested in Mathieu Moonshine and working with Yuji Tachikawa (but not much progress).
- sometimes study non-invertible symmetries, a generalized notion of group symmetry allowing some non-invertibility.
  this talk: [arXiv:2403.20062[hep-th]]

Example of Non-invertible Symmetry

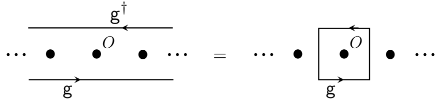
Example (quantum transverse-field Ising chain)

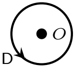
states: $|\overset{s_1}{\bullet}\,\overset{s_2}{\bullet}\cdots\overset{s_n}{\bullet}\rangle$, $s_i = 0$ or 1 (spin-$\frac{1}{2}$)

Hamiltonian: $H = \sum_i (Z_i Z_{i+1} + X_i)$

symmetries $[H, X] = 0$:

- spin flip g
  action on states $g : |\cdots\overset{s_i}{\bullet}\cdots\rangle \mapsto |\cdots\overset{s_i+1(\mathrm{mod}2)}{\bullet}\cdots\rangle$
  g is unitary and invertible $g^{-1} = g^\dagger = g$.
  action on local operators $g : O \mapsto g^\dagger O g$

  $g(O) =$ 

- Kramers-Wannier duality D
  action on states $D : \langle\cdots\overset{t_i}{\bullet}\cdots|D|\cdots\overset{s_i}{\bullet}\cdots\rangle = \prod_i (-1)^{t_i(s_i+s_{i+1})}$
  D is non-invertible, but its dual satisfies $D^\dagger D = \mathbf{1} + g$.
  action on local operators $\mathcal{D} : O \mapsto$ 

  e.g. $\mathcal{D} : Z_i Z_{i+1} \mapsto X_i, X_i \mapsto Z_{i-1} Z_i$.

Background: Bischoff's idea

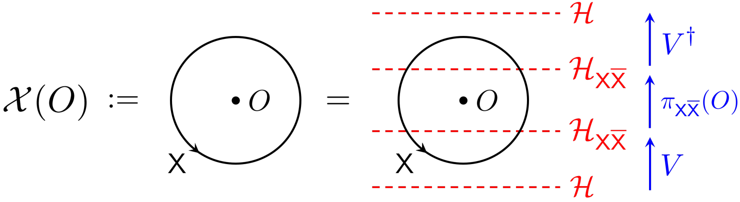
In the context of local conformal net (2d CFT), Bischoff et. al. proved the following [arXiv:1608.00253[math-ph]], [arXiv:2204.14105[math.OA]], ... :
Let us generalize the orbifold of a theory $\mathcal{A}$ by
the action of a finite group on $\mathcal{A}$ by automorphisms
to
the action of a hypergroup on $\mathcal{A}$ by completely positive maps.
Then Galois-theory-like statements hold for local conformal nets.

- In quantum field theory, hypergroups are used to describe the algebra of non-invertible symmetries.
- In quantum information theory, completely positive maps describe quantum operations, processes which can be performed on quantum systems.

$\rightarrow$ We want to introduce Bischoff's idea to physics, in a physicist-friendly language.

Non-invertible Symmetry Acts locally as Quantum Operation

the key idea:



The action of a non-invertible symmetry X on local operators $O \mapsto \mathcal{X}(O)$ has a Stinespring representation $\mathcal{X}(O) = V^\dagger \pi_{X\overline{X}}(O) V$.
$\Rightarrow$ It is a completely positive map.

- We also demonstrated this in the example of the Ising chain.
- This picture is valid not only in 2d CFTs, but also in any-dimensional QFTs with codim-1 non-invertible symmetries.
- more interaction between non-invertible symmetry and quantum information?

(Backup) Stinespring Rep. of Completely Positive Map

quantum operation from a system on $\mathcal{H}_A$ to $\mathcal{H}_B$:

1. Couple the system $\mathcal{H}_A$ with an ancillary system $\mathcal{H}_R$ as $\mathcal{H}_A \otimes \mathcal{H}_R =: \mathcal{H}_E$.
2. Perform unitary quantum gates $\mathcal{H}_E \to \mathcal{H}_E$.
3. Trace out the extra system (perform measurement) as $\mathcal{H}_E \to \mathcal{H}_B$.

This defines a completely positive map from density matrices on $\mathcal{H}_A$ to those on $\mathcal{H}_B$.

More generally,
($B(\mathcal{H})$: the C^*-algebra of bounded linear operators on the Hilbert space $\mathcal{H}$)
A linear map $\mathcal{E} : B(\mathcal{H}_A) \to B(\mathcal{H}_B)$ is completely positive
$\iff$ ${}^\exists\mathcal{H}_E,\ {}^\exists V : \mathcal{H}_A \to \mathcal{H}_E,\ {}^\exists\pi : B(\mathcal{H}_B) \to B(\mathcal{H}_A)$ $*$-hom.,

$$\mathcal{E}(O_B) = V^*\pi(O_B)V \text{ for any } O_B \in B(\mathcal{H}_B).$$

"$\mathcal{E}$ has a Stinespring representation $(\pi, V, \mathcal{H}_E)$."